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	I	II	III	IV
FeO.....	14.58	3.35	16.435	16.13
NiO.....	0.48 ⁵	0.03		0.21
CoO.....	0.06 ⁶			
CaO.....	2.42	4.81	1.758	2.31
BaO.....	none	0.10		
MgO.....	22.67	3.77	22.884	22.42
MnO.....	0.29 ⁷	0.09	0.556	0.18
SrO.....	none	0.04		
Na ₂ O.....	0.87 ⁸	3.29	0.943	0.81
K ₂ O.....	0.21 ⁹	3.02	0.328	0.20
Li ₂ O.....	trace	0.01		
H ₂ O(Ign.).....	0.75 ¹⁰	2.05		0.20
P ₂ O ₅	0.26 ¹¹	0.25		0.03
S.....	1.80 ¹²	0.10	1.839	1.98
Cu.....	0.014 ¹³			
C.....	0.15 ¹⁴	0.03		0.06
Cl.....	0.08 ¹⁵	0.06		
F.....	?	0.10		
CO ₂	?	0.70		
SO ₃		0.02		
Ni, Mn..... }				0.02
Cu, Sn..... }				
	100.04 ₄	100.00	100.00	99.82

¹ Average of 46 determinations

² Average of 42 determinations.

³ Average of 50 determinations.

⁴ Average of 41 determinations.

⁵ Average of 19 determinations.

⁶ Average of 6 determinations.

⁷ Average of 33 determinations

⁸ Average of 49 determinations.

⁹ Average of 44 determinations.

¹⁰ Average of 15 determinations.

¹¹ Average of 44 determinations.

¹² Average of 51 determinations.

¹³ Average of 16 determinations

¹⁴ Average of 8 determinations.

¹⁵ Average of 5 determinations.

The detailed results of these investigations on the chemical and mineralogical constituents of meteorites begun in 1912 under a grant from the National Academy will be printed as a Memoir in the series of *Memoirs of the National Academy*.

ON THE REPRESENTATION OF ARBITRARY FUNCTIONS BY DEFINITE INTEGRALS

By Walter B. Ford

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Presented to the Academy, May 27, 1915

Let $f(x)$ be any function of the real variable x defined and with $|f(x)|$ integrable throughout the interval (a, b) and having limited total fluctuation in the neighborhood of the particular point $x = \alpha$ ($a < \alpha < b$). Then, if $\varphi(n, x - \alpha)$ be a function of the parameter n and of $x - \alpha$ satisfying certain well known conditions the integral

$$I_n(\alpha) = \int_a^b f(x) \varphi(n, x - \alpha) dx \quad (1)$$

will have the property that

$$\lim_{n \rightarrow +\infty} I_n(\alpha) = \frac{f(\alpha - 0) + f(\alpha + 0)}{2}. \quad (2)$$

An important special instance of an integral (1) having the property (2) is presented in the study of the convergence of the Fourier series for $f(\alpha)$, in which case the sum of the first $n + 1$ terms of the series can be put into the form (1) with $a = -\pi$, $b = \pi$ and $\varphi(n, x - \alpha) =$ a certain trigonometric expression.

While the conditions upon $\varphi(n, x - \alpha)$ that will insure (2) have been extensively studied, especially by Du Bois Reymond, Dini, Hobson and Lebesgue, relatively little appears to have been done in the actual determination of such functions, the sole desideratum being the determination of the conditions themselves. In this connection the present paper would point out a noteworthy class of possible functions φ with special emphasis upon the corresponding integrals (1) to which they give rise. Four theorems are established, the second being especially noteworthy in that it shows that to every convergent improper integral of the form

$$\int_0^\infty p(x) dx = k \neq 0,$$

wherein $p(x)$ satisfies very simple conditions, there can be made to correspond a certain integral (1) having the property (2).

The theorems are as follows:

Theorem I: Let $F(x)$ be any single valued function of the real variable x defined for all finite values of x and satisfying the following three conditions:

$$(a) \quad \lim_{x \rightarrow +\infty} F(x) \text{ exists and } = k \neq 0$$

$$(b) \quad F(-x) = -F(x).$$

(c) The derivative $F'(x)$ exists and is such that if we exclude the point $x=0$ by an arbitrarily small interval $(-\epsilon, \epsilon)$, ($\epsilon > 0$), we shall have for all remaining values of x , $|x F'(x)| < A_\epsilon =$ an assignable constant depending only on ϵ . Then, if $f(x)$ be an arbitrary function of the real variable x defined throughout the interval (a, b) , we shall have for any special value α ($a < \alpha < b$)

$$\lim_{n \rightarrow +\infty} \frac{1}{2k} \int_a^b f(x) \frac{d}{dx} F[n(x - \alpha)] dx = \frac{f(\alpha - 0) + f(\alpha + 0)}{2}$$

provided merely that $f(x)$ satisfies suitable conditions (analogous to

those under which the Fourier series for $f(\alpha)$ converges) in the neighborhood of the point $x = \alpha$.

Theorem II: Given any convergent improper integral of the form

$$\int_0^{\infty} p(x) dx = k \neq 0$$

wherein (a) the function $p(x)$ is even, i.e. $p(-x) = p(x)$, and (b) the expression $|x p(x)|$ for all values of x lying outside an arbitrarily small interval surrounding the origin remains less than a constant depending on the interval. Then if $f(x)$ be an arbitrary function of the real variable x defined throughout the interval (a, b) , we shall have for any special value α ($a < \alpha < b$)

$$\lim_{n \rightarrow +\infty} \frac{n}{2k} \int_a^b f(x) p[n(x-\alpha)] dx = \frac{f(\alpha-0) + f(\alpha+0)}{2},$$

provided merely that $f(x)$ satisfies suitable conditions in the neighborhood of the point $x = \alpha$.

Theorem III: Let $F(x)$ be any single valued function of the real variable x which, when considered for positive (negative) values only of x satisfies the following three conditions:

(a) $\lim_{x \rightarrow +\infty} F(x)$ exists and $= k \neq 0$ $\left(\lim_{x \rightarrow -\infty} F(x) \text{ exists and } = -k = 0 \right)$

(b) $F(0) = 0$.

(c) The derivative $F'(x)$ exists and is such that if we exclude the point $x = 0$ by an arbitrarily small interval $(-\epsilon, \epsilon)$, ($\epsilon > 0$), we shall have for all remaining positive (negative) values of x , $|x F'(x)| < A_\epsilon =$ an assignable constant depending only on ϵ .

Then, if $f(x)$ be an arbitrary function of the real variable x defined throughout the interval (a, b) , we shall have for any special value α ($a < \alpha < b$)

$$\lim_{n \rightarrow +\infty} \frac{1}{k} \int_a^b f(x) \frac{d}{dx} F[n(x-\alpha)] dx = f(\alpha+0)$$

$$\left(\lim_{n \rightarrow +\infty} \frac{1}{k} \int_a^\alpha f(x) \frac{d}{dx} F[n(x-\alpha)] dx = f(\alpha-0) \right),$$

provided merely that $f(x)$ satisfies suitable conditions at the right (left) of the point $x = \alpha$.

Theorem IV: Given any convergent improper integral of the form

$$\int_0^{\infty} p(x) dx = k \neq 0 \qquad \left(\int_{-\infty}^0 p(x) dx = k \neq 0 \right)$$

wherein $|x p(x)|$ for all positive (negative) values of x lying outside an arbitrarily small interval to the right (left) of the point $x=0$ remains less than a constant depending on the interval.

Then, if $f(x)$ be an arbitrary function of the real variable x defined throughout the interval (a, b) , we shall have for any special value α ($a < \alpha < b$)

$$\lim_{n \rightarrow +\infty} \frac{n}{k} \int_{\alpha}^b f(x) p[n(x-\alpha)] dx = f(\alpha+0)$$

$$\lim_{n \rightarrow +\infty} \frac{n}{k} \int_a^{\alpha} f(x) p[n(x-\alpha)] dx = f(\alpha-0),$$

provided merely that $f(x)$ satisfies suitable conditions in the neighborhood at the right (left) of the point.

The proof of Theorem I follows directly from the fact (see for example, Dini's *Serie di Fourier* (Pisa, 1880), pp. 119-121) that if $f(x)$ satisfies the indicated conditions there exists the general relation

$$\lim_{n \rightarrow +\infty} \int_a^b f(x) \varphi(n, x-\alpha) dx = \frac{f(\alpha-0) + f(\alpha+0)}{2},$$

whenever $\varphi(n, t)$ is any function of the independent variables n and t satisfying the following three conditions, ϵ always denoting an arbitrarily small positive quantity:

- (I) $\lim_{n \rightarrow +\infty} \int_0^t \varphi(n, t) dt = \begin{cases} -\frac{1}{2} & \text{when } -\epsilon < t < 0 \\ +\frac{1}{2} & \text{when } 0 < t < \epsilon \end{cases}$
- (II) $\left| \int_0^t \varphi(n, t) dt \right| < c_1$ when $-\epsilon < t < \epsilon$, c_1 being a constant (dependent only on ϵ)
- (III) $|\varphi(n, t)| < c_2$ when $\begin{cases} a-\alpha < t < -\epsilon \text{ or} \\ 0 < t < b-\alpha \end{cases}$, c_2 being a constant (dependent only on ϵ)

Theorem II is a corollary of Theorem I.

Theorem III results from the fact (cf. Dini, l.c.) that if the conditions (I), (II), (III) above hold only for the positive (negative) values of t there specified, then, whenever $f(x)$ satisfies suitable conditions at the right (left) of the point $x=\alpha$, we may write

$$\lim_{n \rightarrow +\infty} \int_{\alpha}^b f(x) \varphi(n, x - \alpha) dx = \frac{1}{2} f(\alpha + 0)$$

$$\left(\lim_{n \rightarrow +\infty} \int_a^{\alpha} f(x) \varphi(n, x - \alpha) dx = \frac{1}{2} f(\alpha - 0) \right)$$

Theorem IV is a corollary of Theorem III.

While the forms of representation for an arbitrary function $f(x)$ afforded by the preceding theorems do not, strictly speaking, represent the function in terms of definite integrals, but rather in terms of the limits of such integrals as the parameter n increases to $+\infty$, it is to be observed that the first member of (2) may always be expressed as a convergent series, viz:

$$I_0(\alpha) + \sum_{n=0}^{\infty} [I_{n+1}(\alpha) - I_n(\alpha)]$$

and thus it appears that to every integral (2) obtained by any one of the preceding theorems there corresponds an actual representation of the arbitrary function in series of definite integrals.

THE LYMPHOCYTE AS A FACTOR IN NATURAL AND INDUCED RESISTANCE TO TRANSPLANTED CANCER

By James B. Murphy and John J. Morton

ROCKEFELLER INSTITUTE FOR MEDICAL RESEARCH, NEW YORK

Presented to the Academy, June 22, 1915

Histologically there is a striking resemblance between the series of phenomena which take place about a failing tissue graft in a host of a foreign species, and an homologous cancer graft in an animal with a natural or induced immunity to transplanted cancer. A constant finding in both cases is a local lymphoid reaction which appears early in the process, and lasts till the destruction of the tissue or cancer graft is complete. We have shown in previous communications that the lymphoid tissue is apparently the important factor in the destruction of a tissue graft in an animal of a foreign species. The facts which lead up to this conclusion are, that an organism like the chick embryo, which normally has no defensive agents against the cells of a foreign species, if supplied with adult lymphoid tissue becomes as resistant as the adult animal in this respect. Furthermore when the adult animal is deprived of the major portion of its lymphoid tissue by repeated small doses of X-ray, it loses the ability to destroy the cells of a foreign species and these will live and grow as well as they would in a native host. It